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ANALYTIC GEOMETRY

Length $\overline{P_1P_2}$ of a directed line segment with initial point P_1 and terminal point P_2.

$$d = |\overline{P_1P_2}| = |x_2 - x_1|$$

Distance (d) between two given points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Coordinates of the point P(x,y) which divides the directed line segment $\overline{P_1P_2}$, given the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, and ratio $r = \overline{P_1P} : \overline{PP_2}$

$$x = \frac{x_1 + rx_2}{1 + r} \qquad r \neq -1$$

$$y = \frac{y_1 + ry_2}{1 + r} \qquad r \neq -1$$

Coordinates of the midpoint Pm(x,y) of the directed line segment $\overline{P_1P_2}$, with given end points $P_1(x_1, y_1)$ y $P_2(x_2, y_2)$.

$$x = \frac{x_1 + x_2}{2}$$

$$y = \frac{y_1 + y_2}{2}$$

ANALYTIC GEOMETRY

The slope or angular coefficient (m) of a line is the tangent of its angle of inclination (α).

$$m = tan(\alpha)$$

Slope (m) of the straight line passing through two given points $P_1(x_1, y_1) \wedge P_2(x_2, y_2)$.

$$m = \frac{y_1 - y_2}{x_1 - x_2} \; ; \; x_1 \neq x_2$$

Angle (θ) formed by two straight lines with initial slope m_1 and terminal slope m_2.

$$tan(\theta_1) = \frac{m_2 - m_1}{1 + m_2 m_1} \; ; \; m_2 m_1 \neq -1$$

Necessary and sufficient condition for the parallelism of two given straight lines having slopes m_1 and m_2.

$$m_1 = m_2$$

Necessary and sufficient condition for the perpendicularity of two given straight lines having slopes m_1 and m_2.

$$m_1 m_2 = -1$$

ANALYTIC GEOMETRY

Point–Slope Form of Equation of a Straight Line

$$y - y_1 = m(x - x_1)$$

Slope–Intercept Form of the Equation of a Straight Line

$$y = mx + b$$

Intercept Form of the Equation of a Straight Line

$$\frac{x}{a} + \frac{y}{b} = 1 \; ; a \wedge b \neq 0$$

General Form of the Equation of a Straight Line

$$Ax + By + C = 0$$

Slope: $m = -\frac{A}{B}$

Intercept: $b = -\frac{C}{B}$

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ANALYTIC GEOMETRY

From the general forms, Ax+By+C=0 y A′x+B′y+C′=0 ;
the following relations are necessary and sufficient conditions for

1. PARALLELISM: $-\frac{A}{B} = -\frac{A'}{B'} \Longrightarrow AB' - A'B = 0$
2. PERPENDICULARITY: $AA' + BB' = 0$
3. COINCIDENCE: $A = kA'$; $B = kB'$; $C = kC'$ $(k \neq 0)$
4. INTERSECTION IN ONE AND ONLY ONE POINT: $\frac{A}{B} \neq \frac{A'}{B'} \Longrightarrow AB' - A'B \neq 0$

Normal Form of the equation of a straight line: $x \cdot cos(\theta) + y \cdot sin(\theta) - p = 0$

To find the equation in the normal form of a line defined by the general form Ax+By+C=0 divide each term by: $r = \pm\sqrt{A^2 + B^2}$

The distance "d" of a straight line to a given point: $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

ANALYTIC GEOMETRY

TRANSLATION OF THE COORDINATE AXES

THEOREM. If the coordinate axes are translated to a new origin O′(h,k); The new parallel coordinate axes will be x' and y', generating new coordinates of P(x,y) and P′(x′,y′). Then, the equations of transformation from the old system to the new coordinate system are:

$$x = x' + h$$

$$y = y' + k$$

ANALYTIC GEOMETRY

The circle whose center is the point C(h,k) and whose radius is the constant "r" has as equation (STANDARD FORM):

$$(x-h)^2+(y-k)^2=r^2$$

The circle whose center is at the origin and whose radius is the constant "r" has as equation (CANONICAL FORM):

$$x^2+y^2=r^2$$

If we algebraically develop the equation of the circle in standard form, results the GENERAL FORM:

$$x^2+y^2+Dx+Ey+F=0$$

Where: $D=-2h$; $E=-2k$; $F=h^2+k^2-r^2$

To transform the equation of the circle in GENERAL FORM TO ITS STANDARD FORM, use the completing the square method, obtaining:

$$\left(x+\frac{D}{2}\right)^2+\left(y+\frac{E}{2}\right)^2=\frac{D^2+E^2-4F}{4}$$

Where: $D^2+E^2-4F>0$

Center: $C(h,k)=C\left(-\frac{D}{2},-\frac{E}{2}\right)$

Radius: $r=\frac{\sqrt{D^2+E^2-4F}}{2}$

ANALYTIC GEOMETRY

First Standard Equation of the parabola; vertex at the origin and axis coincident with the x-axis:

$$y^2 = 4px$$

FOCUS: F(p,0)
DIRECTRIX: x=-p
$|\overline{LL'}| = |4p|$

First Standard Equation of the parabola; vertex at the origin and axis coincident with the y-axis:

$$x^2 = 4py$$

FOCUS: F(0,p)
DIRECTRIX: y=-p
$|\overline{LL'}| = |4p|$

Second Standard Equation of the parabola; vertex in V(h,k) and axis parallel to the x-axis:

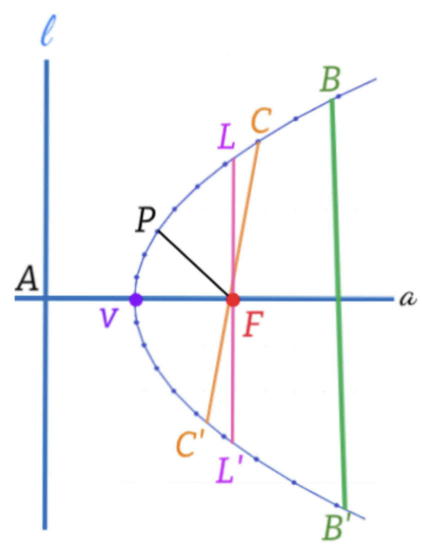
$$(y-k)^2 = 4p(x-h)$$

FOCUS: F(h+p,k)
DIRECTRIX: x=h-p
$|\overline{LL'}| = |4p|$

Second Standard Equation of the parabola; vertex in V(h,k) and axis parallel to the y-axis:

$$(x-h)^2 = 4p(y-k)$$

FOCUS: F(h,k+p)
DIRECTRIX: y=k-p
$|\overline{LL'}| = |4p|$



F = Focus.
l = Directrix.
a = axis of the parabola.
A = point of intersection axis and directrix.
V = vertex.
BB' = chord.
CC' = focal chord.
LL' = latus rectum.
FP = focal radius.

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ANALYTIC GEOMETRY

First Standard Equation of an ellipse; center at the origin and focal axis on the "x" axis.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

FOCI: F(c,0)∧F'(-c,0)
VERTICES: V(a,0)∧V'(-a,0)
ENDS OF MINOR AXIS: A(0,b)∧A'(0,-b)

First Standard Equation of an ellipse; center at the origin and focal axis on the "y" axis.

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

FOCI: F(0,c)∧F'(0,-c)
VERTICES: V(0,a)∧V'(0,-a)
ENDS OF MINOR AXIS: A(b,0)∧A'(-b,0)

For both cases:

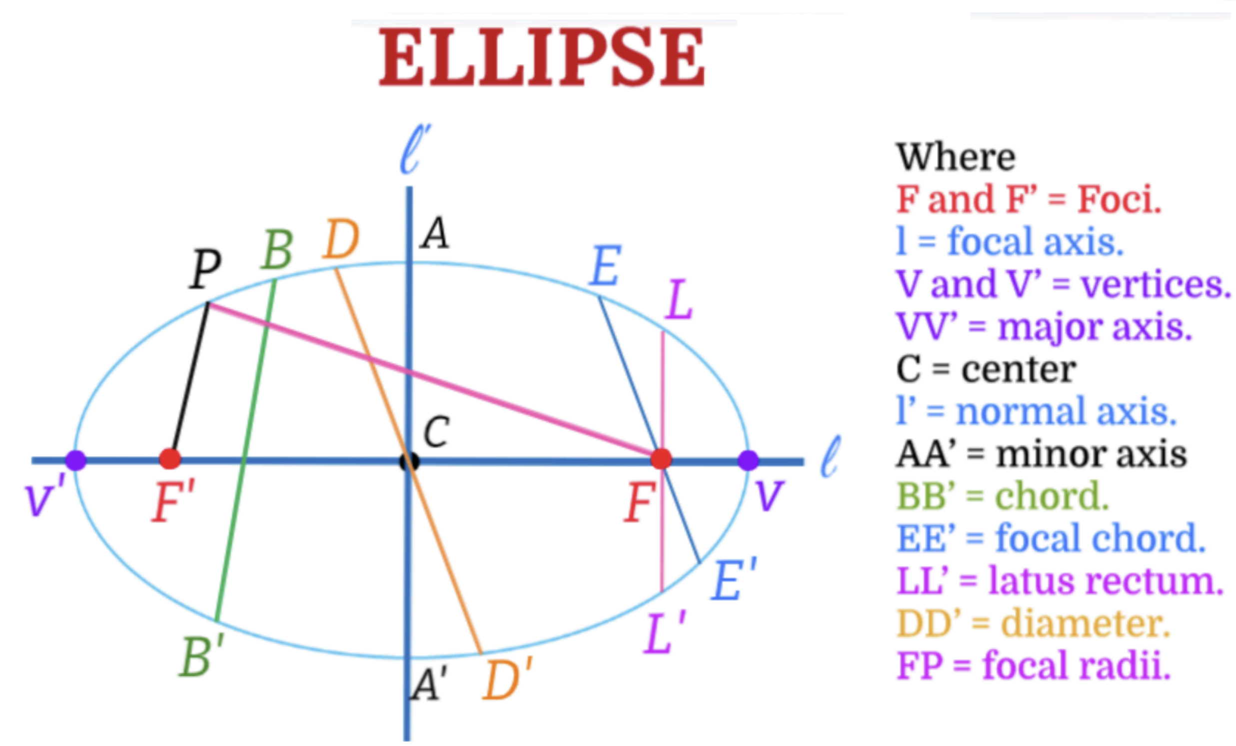
Eccentricity<1

$$e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}$$

$$c^2 = a^2 - b^2$$

Latus Rectum

$$|\overline{LL'}| = \frac{2b^2}{a}$$



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ANALYTIC GEOMETRY

Second Standard Equation of an ellipse; center at C(h,k) and focal axis on the "x" axis.

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

FOCI: F(h+c,k)∧F'(h−c,k)
VERTICES: V(h+a,k)∧V'(h−a,k)
ENDS OF MINOR AXIS: A(h,k+b)∧A'(h,k−b)

Second Standard Equation of an ellipse; center at C(h,k) and focal axis on the "y" axis.

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

FOCI: F(h,k+c)∧F'(h,k−c)
VERTICES: V(h,k+a)∧V'(h,k−a)
ENDS OF MINOR AXIS: A(h+b,k)∧A'(h−b,k)

For both cases:

Eccentricity<1

$$e=\frac{c}{a}=\frac{\sqrt{a^2-b^2}}{a}$$

Latus Rectum

$$|\overline{LL'}|=\frac{2b^2}{a}$$

$$c^2=a^2-b^2$$

A quadratic equation in the variables x and y, lacking the xy term, is:

$$Ax^2+Cy^2+Dx+Ey+F=0$$

If A and C have the same sign, the equation represents an ellipse, a single point or does not represent any locus.

NOTE: To obtain the standard form from the general form, complete the square. To obtain the general form from the standard form, develop algebraically.

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ANALYTIC GEOMETRY

First Standard Equation of an hyperbola; center at the origin and focal axis on the "x" axis.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

FOCI: F(c,0)∧F'(-c,0)
VERTICES: V(a,0)∧V'(-a,0)
CONJUGATE AXIS: A(0,b)∧A'(0,-b)

First Standard Equation of an hyperbola; center at the origin and focal axis on the "y" axis.

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

FOCI: F(0,c)∧F'(0,-c)
VERTICES: V(0,a)∧V'(0,-a)
CONJUGATE AXIS: A(b,0)∧A'(-b,0)

For both cases:

Eccentricity>1

$$e=\frac{c}{a}=\frac{\sqrt{a^2+b^2}}{a}$$

Latus Rectum

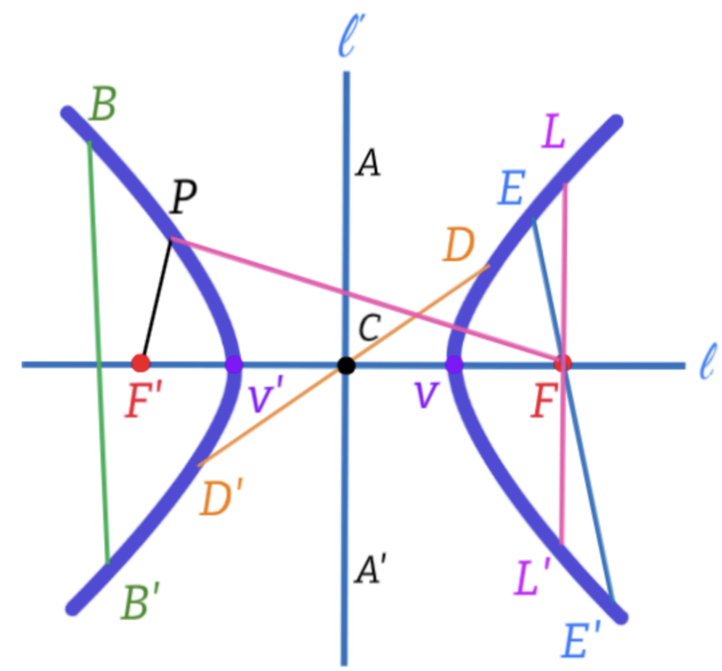
$$\left|\overline{LL'}\right|=\frac{2b^2}{a}$$

$$c^2=a^2+b^2$$

Asymptote equations

bx-ay=0
bx+ay=0

HYPERBOLA



Where
F and F' = Foci.
l = focal axis.
V and V' = vertices.
VV' = transverse axis
C = center.
l' = normal axis.
AA' = conjugate axis
BB' = chord.
EE' = focal chord.
LL' = latus rectum.
DD' = diameter.
FP = focal radii.

ANALYTIC GEOMETRY

Second Standard Equation of a hyperbola; center at C(h,k) and focal axis on the "x" axis.

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

FOCI: F(h+c,k)∧F'(h−c,k)
VERTICES: V(h+a,k)∧V'(h−a,k)
CONJUGATE AXIS: A(h,k+b)∧A'(h,k−b)

Second Standard Equation of a hyperbola; center at C(h,k) and focal axis on the "y" axis.

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

FOCI: F(h,k+c)∧F'(h,k−c)
VERTICES: V(h,k+a)∧V'(h,k−a)
CONJUGATE AXIS: A(h+b,k)∧A'(h−b,k)

For both cases:

Eccentricity>1

$$e=\frac{c}{a}=\frac{\sqrt{a^2+b^2}}{a}$$

Latus Rectum

$$|\overline{LL'}|=\frac{2b^2}{a}$$

$$c^2=a^2+b^2$$

Asymptote equations

$$y-k=\pm\frac{b}{a}(x-h)$$

A quadratic equation in the variables x and y, lacking the xy term, is:

$$Ax^2+Cy^2+Dx+Ey+F=0$$

If A and C differ in sign, the equation represents a hyperbola or a pair of lines that intersect.
NOTE: To obtain the standard form from the general form, complete the square. To obtain the general form from the standard form, develop algebraically.