



# LOGARITHMS

## LOGARITHM

The **logarithm** base “b” of “x” is the **exponent** (y) to which the base (b) must be raised to obtain the number (x); hence:

$$\log_b(x) = y \iff b^y = x$$

Diagram illustrating the logarithmic equation  $\log_b(x) = y \iff b^y = x$ . The base (b) is labeled as "Base", the argument (x) is labeled as "Argument", and the exponent (y) is labeled as "Exponent".

## PROPERTIES OF LOGARITHMS

1. Logarithms of negative numbers do not exist in the system of real numbers.
2. The logarithm of zero is undefined.
3.  $\log_b(1) = y \iff b^y = 1$
4.  $\log_b(b) = 1 \iff b^1 = b$
5.  $\log_b(b^n) = n \iff b^n = b^n$
6.  $\log_b(x^n) = n \cdot \log_b(x)$
7.  $\log_b(\sqrt[n]{x}) = \log_b(x^{\frac{1}{n}}) = \frac{1}{n} \cdot \log_b(x)$
8.  $\log_b(\sqrt[n]{x^m}) = \log_b(x^{\frac{m}{n}}) = \frac{m}{n} \cdot \log_b(x)$
9.  $\log_b(x \cdot y) = \log_b(x) + \log_b(y)$
10.  $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
11. Logarithm change of base rule:

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

$$\log_a(b) = \frac{1}{\log_b(a)}$$



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**EXAMPLE:** Find the value of the following logarithm:

$$\log_2(8) =$$

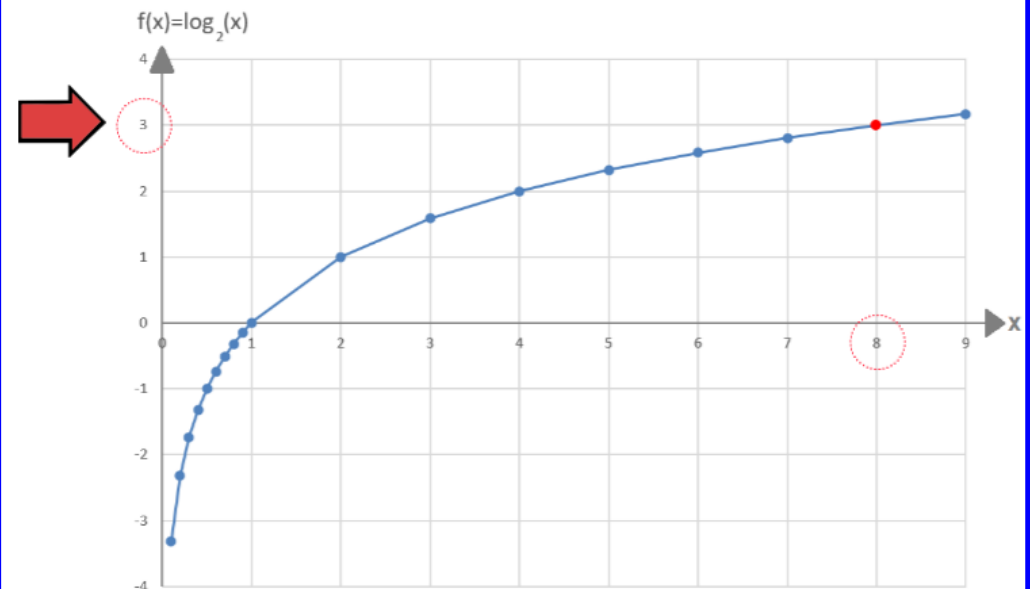
**SOLUTION:** 1) Applying  $\log_b(x) = y \Leftrightarrow b^y = x$

$$\log_2(8) = y \Leftrightarrow 2^y = 8$$

$$\log_2(8) = 3 \Leftrightarrow 2^3 = 8$$

$$\log_2(8) = \underline{3}$$

$$\log_2(8) = \underline{3}$$



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EXAMPLE: Simplify the following expression:

$$4\log_3(x) + \log_3(x) - \frac{\log_3(x)}{2} =$$

SOLUTION: 1) Applying

$$\log_b(x^n) = n \cdot \log_b(x)$$

$$= \log_3(x^4) + \log_3(x) - \frac{\log_3(x)}{2} =$$

$$= \log_3(x^4) + \log_3(x) - \frac{1}{2}\log_3(x) =$$

$$= \log_3(x^4) + \log_3(x) - \log_3(x^{\frac{1}{2}}) =$$

2) Applying

$$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$$

$$= \log_3(x^4) + \log_3(x) - \log_3(x^{\frac{1}{2}}) =$$

$$= \log_3(x^4 \cdot x) - \log_3(x^{\frac{1}{2}}) =$$

$$= \log_3(x^5) - \log_3(x^{\frac{1}{2}}) =$$

3) Applying

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$= \log_3\left(\frac{x^5}{x^{\frac{1}{2}}}\right) =$$

$$= \log_3(x^5 \cdot x^{-\frac{1}{2}}) =$$

$$= \log_3(x^{5-\frac{1}{2}}) =$$

$$= \log_3(x^{\frac{9}{2}}) =$$

$$= \log_3(\sqrt{x^9})$$

