INEQUALITIES AND INEQUATIONS

TEXAN GLOBAL SCHOOL
Global Online Learning

DEFINITION: An "inequality" is defined as a relationship between two expressions that are not equal; hence, one of the expressions is said to be greater or less than the other. The symbols used are $>, <, \leq, \geq$.

PROPERTIES OF INEQUALITIES

1) **ADDITIVE:**

$$\text{If } \boxed{a > b} \Rightarrow \boxed{a + c > b + c}$$

$$\text{If } \boxed{a > b} \Rightarrow a + (-c) > b + (-c) \Rightarrow \boxed{a - c > b - c}$$

2) **MULTIPLICATIVE:**

$$\text{If } \boxed{a > b} \wedge c > 0 \Rightarrow \boxed{a \cdot c > b \cdot c}$$

$$\text{If } \boxed{a > b} \wedge c > 0 \Rightarrow a \cdot \left(\frac{1}{c}\right) > b \cdot \left(\frac{1}{c}\right) \Rightarrow \boxed{\frac{a}{c} > \frac{b}{c}}$$

$$\text{If } \boxed{a > b} \wedge c < 0 \Rightarrow \boxed{a \cdot c < b \cdot c}$$

$$\text{If } \boxed{a > b} \wedge c < 0 \Rightarrow a \cdot \left(\frac{1}{c}\right) < b \cdot \left(\frac{1}{c}\right) \Rightarrow \boxed{\frac{a}{c} < \frac{b}{c}}$$

3) $\text{If } \boxed{a > b}, \boxed{c > d} \Rightarrow \boxed{a + c > b + d}$

$\text{If } \boxed{a > b}, \boxed{c > d} \Rightarrow \boxed{a \cdot c > b \cdot d}$

4) **TRANSITIVE:** $\text{If } \boxed{a > b} \wedge \boxed{b > c} \Rightarrow \boxed{a > c}$

5) ***If*** $a \wedge b$ **are both positive,** $\boxed{a > b} \wedge n \in \mathbb{N}, \Rightarrow \boxed{a^n > b^n}$.

6) ***If*** $a \wedge b$ **are both positive,** $\boxed{a > b} \wedge n \in \mathbb{N}, \Rightarrow \boxed{\sqrt[n]{a} > \sqrt[n]{b}}$.

7) ***If*** $a \wedge b$ **are both positive,** $\boxed{a > b} \wedge n \in \mathbb{N}, \Rightarrow \boxed{a^{-n} < b^{-n}}$.

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EXAMPLE: Find the solution set of the following inequation:

$$x + 1 \leq 2x - 4$$

SOLUTION: 1) Applying additive property:

$x + 1 - 1 \leq 2x - 4 - 1$

$x \leq 2x - 5$

$x - 2x \leq 2x - 5 - 2x$

$-x \leq -5$

2) Applying multiplicative property:

$(-1)(-x) \geq (-5)(-1)$

$x \geq 5$

The solution set is:

$$[5, +\infty\rangle = \{x \mid x \geq 5\}$$

VERIFY:

If $x = 11 \Rightarrow (11) + 1 \leq 2(11) - 4$

$11 + 1 \leq 22 - 4$

$12 \leq 18$ ✓

If $x = 0 \Rightarrow (0) + 1 \leq 2(0) - 4$

$0 + 1 \leq 0 - 4$

$1 \leq -4$ ✗

EXAMPLE: Find the solution region of the following inequality:

$$y + 2 < x^2 - 1$$

SOLUTION: 1) Isolate "y" :

$y + 2 - 2 < x^2 - 1 - 2$

$y < x^2 - 3$

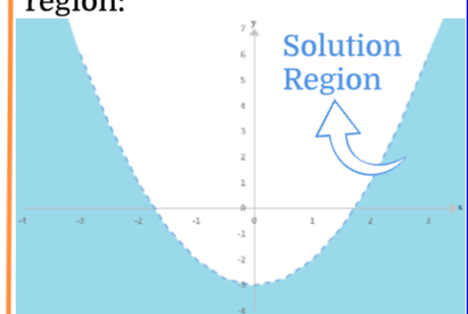
2) Replace inequality symbol with equality:

$y = x^2 - 3$

3) Tabulate:

x	$y = x^2 - 3$	P(x,y)
-3	$y = (-3)^2 - 3 = 6$	P(-3,6)
-2	$y = (-2)^2 - 3 = 1$	P(-2,1)
-1	$y = (-1)^2 - 3 = -2$	P(-1,-2)
0	$y = (0)^2 - 3 = -3$	P(0,-3)
1	$y = (1)^2 - 3 = -2$	P(1,-2)
2	$y = (2)^2 - 3 = 1$	P(2,1)
3	$y = (3)^2 - 3 = 6$	P(3,6)

4) Plot and identify solution region:



If $P(0,0) \Rightarrow x = 0 \wedge y = 0$:

$(0) + 2 < (0)^2 - 1$

$2 < -1$ ✗