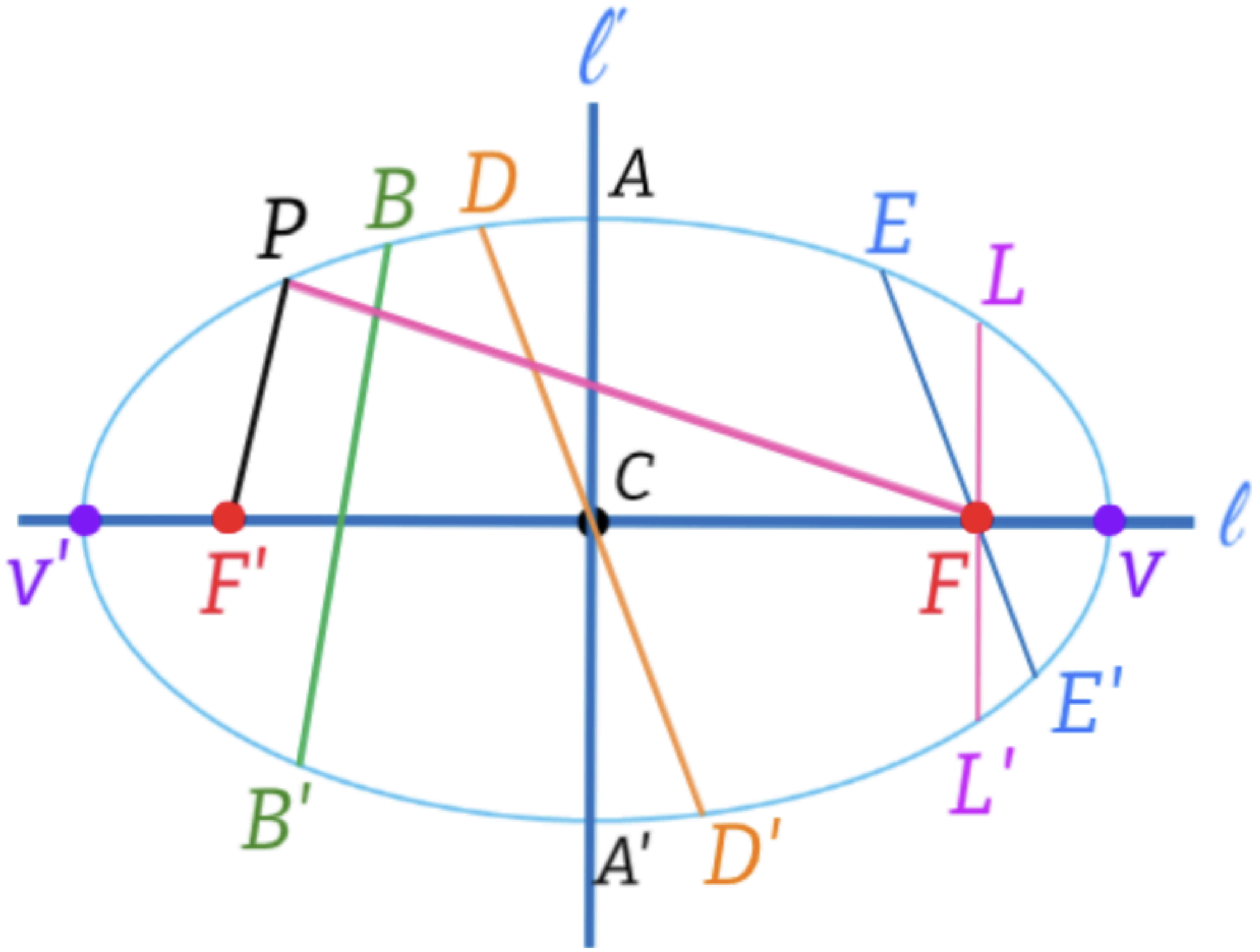
ELLIPSE



Where
F and F' = Foci.
l = focal axis.
V and V' = vertices.
VV' = major axis.
C = center
l' = normal axis.
AA' = minor axis
BB' = chord.
EE' = focal chord.
LL' = latus rectum.
DD' = diameter.
FP = focal radii.

ELLIPSE

KNOWLEDGE FOR THE WORLD
TEXAN GLOBAL SCHOOL
Global Online Learning

FIRST STANDARD EQUATION OF AN ELLIPSE

THEOREM. Equation of an ellipse with center at the origin and focal axis on the “x” axis:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Foci in F(c,0) and F'(-c,0). Vertices (ends of the major axis) in V(a,0) and V'(-a,0). Ends of the minor axis are A(0,b) and A'(0,-b).

Equation of an ellipse with center at the origin and focal axis on the “y” axis:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Foci in F(0,c) and F'(0,-c). Vértices (ends of the major axis) in V(0,a) and V'(0,-a). Ends of the minor axis are A(b,0) and A'(-b,0).

Where:

- “a” is the length of the semi-major axis.
- “b” is the length of the semi-minor axis.
- “c” is a positive constant that represents the distance from the center to the focus.
- Relation between a, b, and c:

$$c^2 = a^2 - b^2$$

- Distance between the two foci is 2c and length of the major axis equal to 2a.
- Latus rectum: $|\overline{LL'}| = \frac{2b^2}{a}$
- Eccentricity:

$$e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}$$

www.texanglobalschool.com

ELLIPSE

SECOND STANDARD EQUATION OF THE ELLIPSE

THEOREM. Equation of an ellipse with center at C(h,k) and axis parallel to the x-axis:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Foci in F(h+c,k) and F'(h-c,k). Vertices (ends of the major axis) in V(h+a,k) and V'(h-a,k). Ends of the minor axis are A(h,k+b) and A'(h,k-b).

Equation of an ellipse with center at C(h,k) and axis parallel to the y-axis:

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Foci in F(h,k+c) and F'(h,k-c). Vertices (ends of the major axis) in V(h,k+a) and V'(h,k-a). Ends of the minor axis are A(h+b,k) and A'(h-b,k).

Where:

- "a" is the length of the semi-major axis.
- "b" is the length of the semi-minor axis.
- "c" is a positive constant that represents the distance from the center to the focus.
- Relation between a, b, and c:

$$c^2=a^2-b^2$$

- Distance between the two foci is 2c and length of the major axis equal to 2a.
- Latus rectum: $|\overline{LL'}|=\frac{2b^2}{a}$
- Eccentricity:

$$e=\frac{c}{a}=\frac{\sqrt{a^2-b^2}}{a}$$

ELLIPSE

EXAMPLE: Find the equation of the ellipse whose foci are the points (2,0) , (-2,0); and its eccentricity is 2/3.

SOLUTION: 1) Foci F(2,0) and F'(-2,0), on the "x" axis.

Since the foci are in F(c,0) and F'(-c,0), hence c=2.

2) Midpoint is the origin.

$$P_m(x,y) = P_m\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$P_m(x,y) = P_m\left(\frac{2-2}{2}, \frac{0+0}{2}\right) = P_m(0,0)$$

3) Eccentricity: $e = \frac{2}{3} = \frac{c}{a}$

⇒ c=2 ; a=3

4) Vertices (ends of the major axis) V(a,0) and V'(-a,0) => V(3,0) and V'(-3,0).

5) Since a=3, c=2; finding "b":

$$c^2 = a^2 - b^2$$
$$(2)^2 = (3)^2 - b^2$$
$$4 = 9 - b^2$$
$$b^2 = 9 - 4$$
$$b^2 = 5$$
$$b = \sqrt{5}$$

6) Ends of the minor axis A(0,b) and A'(0,-b); hence:

$$A(0,\sqrt{5}) \wedge A'(0,-\sqrt{5})$$

7) Since a=3 , b= $\sqrt{5}$, the equation of an ellipse with center at the origin and focal axis on the "x" axis is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{(3)^2} + \frac{y^2}{(\sqrt{5})^2} = 1$$
$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$ ✓

8) Latus rectum:

$$|\overline{LL'}| = \frac{2b^2}{a} = \frac{2(\sqrt{5})^2}{3} =$$
$$|\overline{LL'}| = \frac{2(5)}{3} = \frac{10}{3}$$

Graphing: