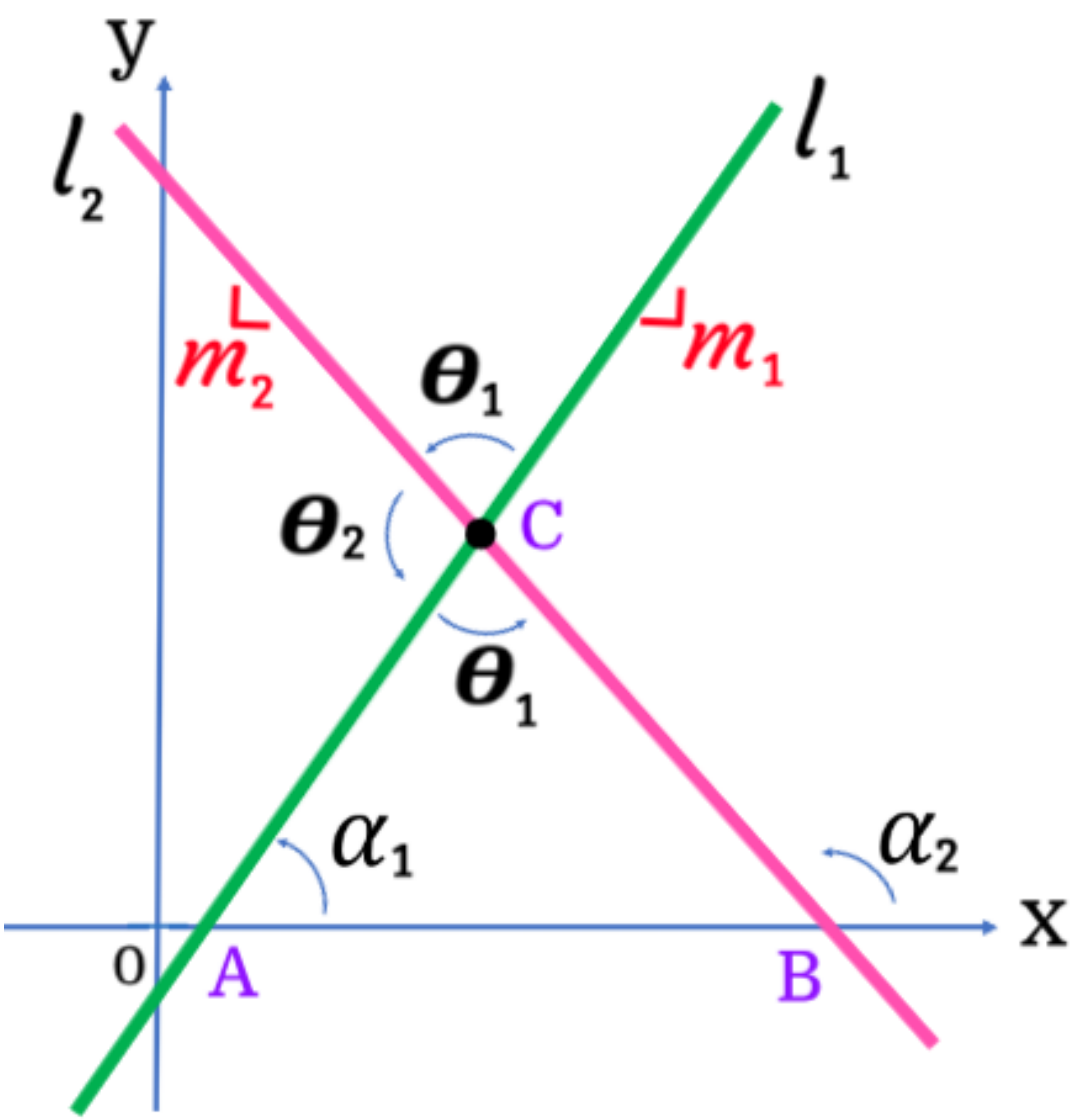
KNOWLEDGE FOR THE WORLD

TEXAN GLOBAL SCHOOL

Global Online Learning

ANGLE BETWEEN TWO LINES

Given two lines defined as l_1 and l_2:



Finding $\boldsymbol{\theta_1}$:

$$\alpha_2 = \alpha_1 + \theta_1$$

The angle between two lines is:

$$\theta_1 = \alpha_2 - \alpha_1$$

Applying tangents:

$$tan(\theta_1) = tan(\alpha_2 - \alpha_1)$$

Trigonometric Identity:

$$tan(A \pm B) = \frac{tan(A) \pm tan(B)}{1 \mp tan(A)tan(B)}$$

Obtaining:

$$tan(\theta_1) = \frac{tan(\alpha_2) - tan(\alpha_1)}{1 + tan(\alpha_2)tan(\alpha_1)}$$

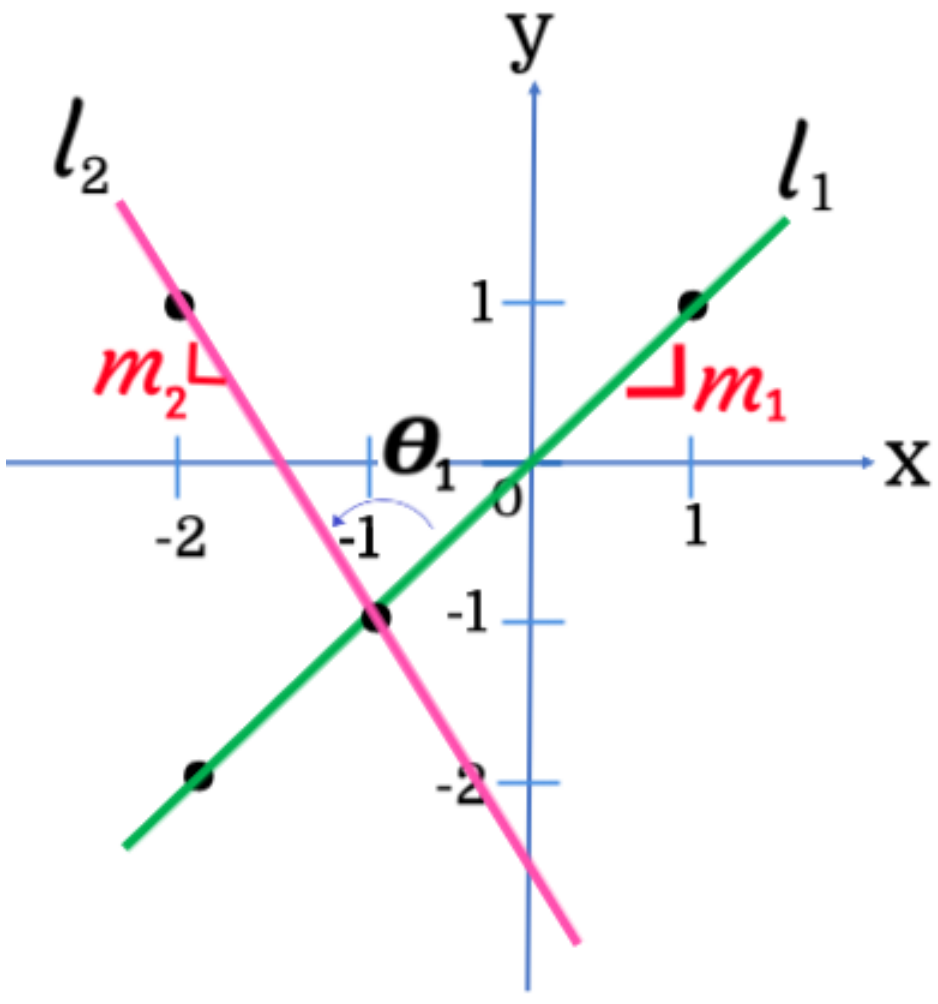
Slope or angular coefficient (m): $m = tan(\alpha)$

Angle between two lines:

$$tan(\theta_1) = \frac{m_2 - m_1}{1 + m_2 m_1}$$

EXAMPLE: Find the angle $\boldsymbol{\theta_1}$ formed by the lines l_1 and l_2. The points $P_1(1,1)$ and $P_2(-2,-2)$ belong to the line l_1. The points $P_1(-2,1)$ and $P_2(-1,-1)$ belong to the line l_2.

SOLUTION:



Slope given two points:

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$P_1(1,1) \wedge P_2(-2,-2)$:

$$m_1 = \frac{1-(-2)}{1-(-2)} = \frac{1+2}{1+2} = \frac{3}{3} = 1$$

$P_1(-2,1) \wedge P_2(-1,-1)$:

$$m_2 = \frac{1-(-1)}{-2-(-1)} = \frac{1+1}{-2+1} = \frac{2}{-1} = -2$$

Angle between two lines:

$$tan(\theta_1) = \frac{m_2 - m_1}{1 + m_2 m_1}$$

$$tan(\theta_1) = \frac{-2-1}{1+(-2)(1)} \qquad tan(\theta_1) = \frac{-3}{-1}$$

$$tan(\theta_1) = \frac{-3}{1-2} \qquad tan(\theta_1) = 3$$

$$\theta_1 = arctan(3) \approx 71.56^o$$